J. Phys. A: Math. Gen. **28** (1995) 1467–1468. Printed in the UK

**ADDENDUM**

# Pseudoclassical mechanics and its solution

Georg Junker and Stephan Matthiesen

Institut für Theoretische Physik, Universität Erlangen-Nürnberg, Staudtstr. 7, 91058 Erlangen, Germany

Received 16 November 1994

**Abstract.** In a recent letter (1994 *J. Phys. A: Math. Gen.* **27** L751) we discussed the properties of a supersymmetric classical system. The purpose of this addendum is to point out that our previous results can be generalized to pseudoclassical models of which the supersymmetric system is a special case.

Pseudomechanics or pseudoclassical mechanics, a notion which has been introduced in 1976 by Casalbuoni [1], describes classical systems which in addition to their usual bosonic (commuting) also have fermionic (anticommuting) degrees of freedom. In fact, pseudoclassical mechanics can be understood as the classical limit (in the sense $\hbar \to 0$) of a quantum system having both kinds of degrees of freedom [2]. This type of classical mechanics has been of particular interest because of its capability for describing a spin degree of freedom on the classical level [3].

The simplest non-trivial example of pseudoclassical mechanics is given by the Lagrangian [2]

$$L := \tfrac{1}{2}\dot{x}^2 - V_1(x) + \tfrac{\mathrm{i}}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - V_2(x)\bar{\psi}\psi \tag{1}$$

where, as in [4], $x$ denotes a bosonic degree of freedom and $\psi$ and $\bar{\psi}$ are two independent fermionic degrees of freedom. Furthermore, $V_1$ and $V_2$ are real-valued functions and the overdot denotes differentiation with respect to time.

The purpose of this addendum is to point out that the solutions of the equations of motion corresponding to (1),

$$\dot{\bar{\psi}} = \mathrm{i}V_2(x)\bar{\psi} \qquad \dot{\psi} = -\mathrm{i}V_2(x)\psi \qquad \ddot{x} = -V_1'(x) - V_2'(x)\bar{\psi}\psi \tag{2}$$

can be obtained in a way similar to our previous approach [4], where we have considered the special case $V_1 = \frac{1}{2}V^2$, $V_2 = \frac{\mathrm{d}V}{\mathrm{d}x}$, which is related to an additional supersymmetry of the system characterized by (1).

As in [4] we make the ansatz $x(t) = x_{\mathrm{qc}}(t) + q(t)\bar{\psi}_0\psi_0$, where $x_{\mathrm{qc}}(t)$ and $q(t)$ are real-valued functions of time and $\bar{\psi}_0 := \bar{\psi}(0)$, $\psi_0 := \psi(0)$. The solutions of (2) are then given by

$$\bar{\psi}(t) = \bar{\psi}_0 \exp\left\{\mathrm{i}\int_0^t \mathrm{d}\tau\, V_2\big(x_{\mathrm{qc}}(\tau)\big)\right\} \qquad \psi(t) = \psi_0 \exp\left\{-\mathrm{i}\int_0^t \mathrm{d}\tau\, V_2\big(x_{\mathrm{qc}}(\tau)\big)\right\} \tag{3}$$

$$q(t) = \frac{\dot{x}_{\mathrm{qc}}(t)}{\dot{x}_{\mathrm{qc}}(0)}\left[q(0) + \frac{\dot{x}_{\mathrm{qc}}(0)}{2}\int_0^t \mathrm{d}\tau\, \frac{F - V_2\big(x_{\mathrm{qc}}(\tau)\big)}{E - V_1\big(x_{\mathrm{qc}}(\tau)\big)}\right]. \tag{4}$$

Here $E, F \in \mathbb{R}$ are constants of integration related to the conserved energy [4] $\mathcal{E} = E + F\bar{\psi}_0\psi_0$ of the system (1). Here we would like to point out that in equation (13) of [4] the prefactor $\dot{x}_{qc}(0)$ in front of the integral is missing. Compare with (4) above.

Again we find, as in [4], that the solutions (3), (4) are expressible in terms of the so-called quasi-classical solution $x_{qc}(t)$ of

$$\dot{x}_{qc}^2 = 2\big[E - V_1(x_{qc})\big]. \tag{5}$$

We also note that for a potential $V_1$ which is bounded from below (without loss of generality we may assume in this case $V_1(x_{qc}) \geqslant 0$ for all $x_{qc} \in \mathbb{R}$ and hence we can set $V^2 := 2V_1$), the discussion of [4] starting with equation (14) can be carried over with all its interesting consequences for the path integral of the quantum system corresponding to (1).

## Acknowledgment

We would like to thank Professor Roberto Casalbuoni for pointing out to us the close connection between pseudoclassical and supersymmetric classical systems.

## References

[1] Casalbuoni R 1976 *Nuovo Cimento* A **33** 115

[2] Casalbuoni R 1976 *Nuovo Cimento* A **33** 389

[3] Berezin F A and Marinov M S 1975 *JETP Lett.* **21** 678
Casalbuoni R 1976 *Phys. Lett.* **62B** 49
Barducci A, Casalbuoni R and Lusanna L 1976 *Nuovo Cimento* A **35** 377
Brink L, Deser S, Zumino B, Di Vecchia and Howe P 1976 *Phys. Lett.* **64B** 435
Berezin F A and Marinov M S 1977 *Ann. Phys., NY* **104** 336
Barducci A, Bordi F and Casalbuoni R 1981 *Nuovo Cimento* B **64** 287
Lahiri A, Roy P K and Bagchi B 1990 *Int. J. Mod. Phys.* A **5** 1383

[4] Junker G and Matthiesen S 1994 *J. Phys. A: Math. Gen.* **27** L751